TRINITY COLLEGE

Year 11 Mathematics Specialist
Test 3 2022

Calculator Assumed
Circle Geometry & Proof

STUDENT'S NAME MARKING KEY [KRISZYK]

DATE: Wednesday 11th May **TIME:** 50 minutes **MARKS:** 43

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser.
Special Items: Scientific Calculator

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (3 marks)

Prove by contradiction the statement "No integers a and b exist for which $24a+12b=1$"

Suppose 24a and 12b can add to give 1.

$[24a + 12b = 1] \div 12$.

$\therefore 2a + b = \frac{1}{12}$

Since the sum of integers a and b is a fraction and the sum of two integers cannot yield a non-integer

$\therefore$ statement cannot be proven false

$\therefore$ statement is true.

2. (8 marks)

(a) For each of the following statements, state whether they are always true or sometimes false. Support each answer with an example.

(i) If $P \Rightarrow Q$ then it follows that $Q \Rightarrow P$. [2]

False

If $x = 2$ then $x^2 = 4$ but if $x^2 = 4$ the $x = \pm 2$

(ii) If $P \Leftrightarrow Q$, then it follows that $Q \Rightarrow P$ and $P \Rightarrow Q$. [2]

True

If $2x = 6$ the $x = 3$

(iii) If $P \Rightarrow Q$ then it follows that $\overline{P} \Rightarrow \overline{Q}$. [2]

False

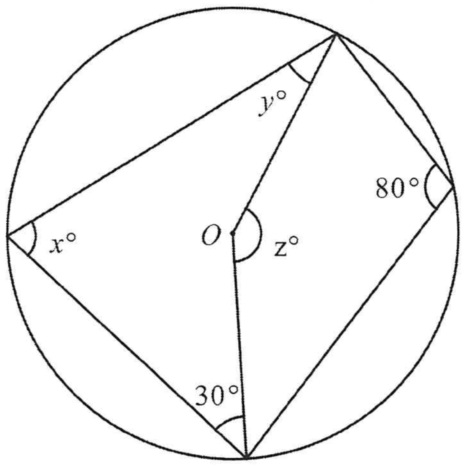
If $x = 2$ the $x^2 = 4$ but if $x \neq 2$ x^2 can still equal 4 when $x = -2$.

(b) If $B \Rightarrow A$ is a true statement, write a statement which relates to A and B which will be **always** true. [2]

$\overline{A} \Rightarrow \overline{B}$

3. (3 marks)

In the diagram below determine the values of x, y and z.



$x = 100°$

$y = 70°$

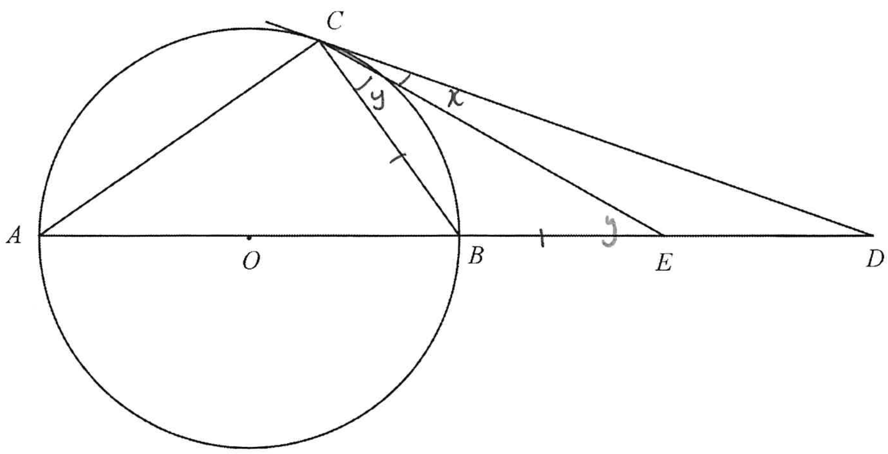
$z = 200°$

allow $z = 160°$

$y = 30°$

4. (3 marks)

Triangle ABC is inscribed in a circle with AB as a diameter. The tangent at C meets AB produced at D, the point E is on the line BD such that BE = BC. Given that $\angle DCE = x^\circ$ and $\angle BCE = y^\circ$.



Calculate, in terms of x and y only, the angles CEB, CBA and CAB.

$\angle CEB = y$

$\angle CBA = 2y$ or $90 - x - y$

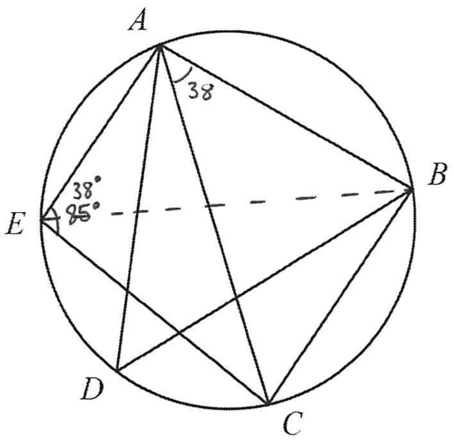
$\angle CAB = 90 - 2y$ or $90 - (90 - x - y)$

$= x + y$

5. (7 marks)

(a) In the diagram below $\angle AEC = 85°$ and $\angle BAC = 38°$. Determine the size of $\angle ADB$.

(Show all relevant angles on the diagram below) [3]

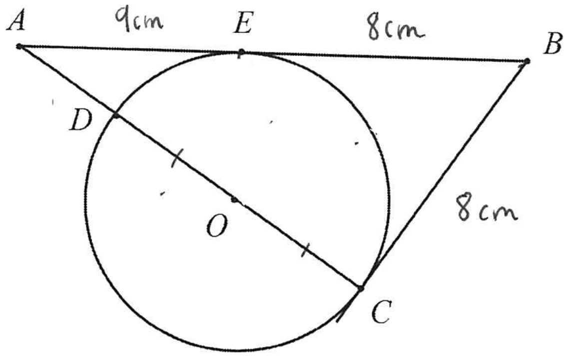


$\angle ADB = 47°$ ✓

Draws line EB ✓

Calculates $\angle AEB$ ✓

(b) In the diagram shown below, not drawn to scale, a circle with centre O has tangents at E and C that meet at B. If the length of BC is 8 cm and the length of AE is 9 cm, determine the length of DC. [4]



$EB = 8\text{ cm}$ ✓

From $\triangle ABC$: $AC^2 = 17^2 + 8^2$

$AC = 15$ ✓

$AC \times AD = AE^2$ (square of tangent = product of intercepts)

$\therefore 15 \times AD = 9^2$

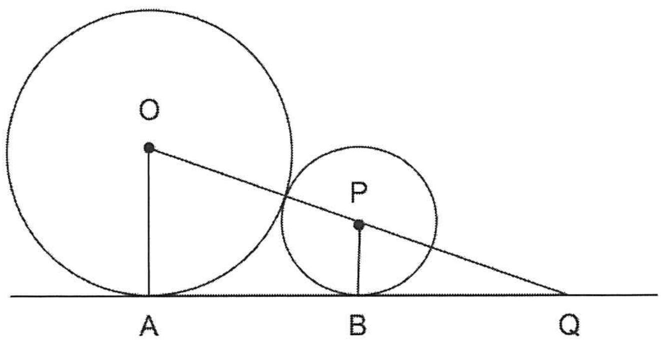
$AD = 5.4\text{ cm}$ ✓

$\therefore DC = 15 - 5.4$

$= 9.6\text{ cm}$ ✓

6. (6 marks)

Two circles are tangent to a line and to each other, as shown in the diagram below. The radius of the larger circle is twice the radius of the smaller circle.



(a) Prove that the triangles AOQ and BPQ are similar. [2]

$\angle OAQ = \angle PBQ = 90°$ (tangent makes ⊥ with radius)

$\angle OQA = \angle PQB$ (common)

$\Delta AOQ \sim \Delta BPQ$ (AA) ✓✓

(b) Show that $PQ = 3r$ where r is the radius of the smaller circle. [2]

$PB = r$

$OA = 2r$

$\therefore OP = 3r$ ✓

By similarity $\frac{OQ}{PQ} = \frac{OA}{PB} = \frac{2r}{r} = 2$

$\therefore OQ = 2PQ$ hence $PQ = OP = 3r$ ✓

(c) Determine the radius of the smaller circle given that AB = 20 cm. [2]

$BQ = AB = 20$

$3r^2 = r^2 + 20^2$ ✓

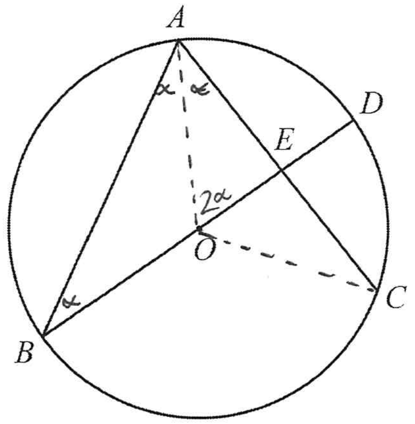
$8r^2 = 400$

$r^2 = 50$

$r = 5\sqrt{2}$ ✓

7. (7 marks)

Consider the diagram below. ΔABC is isosceles with $AB = AC$ and BOD is a diameter where O is the centre of the circle.



Prove $\angle AED = 3 \times \angle ABD$.

In ΔOAB and ΔOAC

OA is common

$OB = OC$ (both radii)

$AB = AC$ (given)

$\therefore \Delta OAB \cong \Delta OAC$ (SSS)

let $\angle OAB = \alpha$

$\Rightarrow \angle OBA = \alpha$ (base $\angle$ of isosceles)

$\therefore \angle AOD = 2\alpha$ (ext $\angle$ = sum of opp int $\angle$s)

$\angle OAC = \angle OAB = \alpha$ (corresponding $\angle$s of cong. Δs)

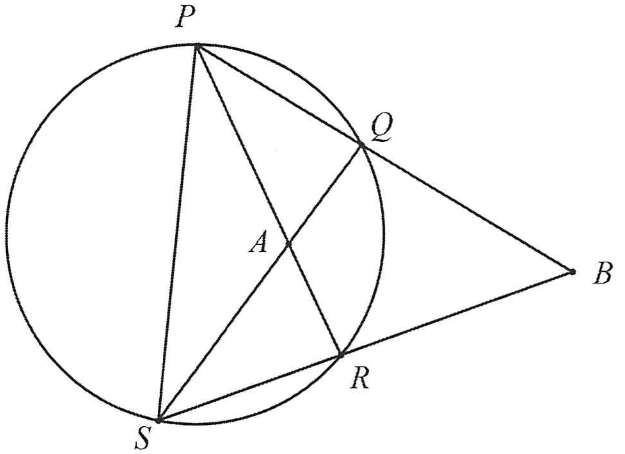
$\therefore \angle AED = 3\alpha$ (ext. $\angle$ = sum of opp int $\angle$s)

Hence $\angle AED = 3\angle ABD$.

Q.E.D.

8. (6 marks)

The points P, Q, R and S lie on a circle of radius r. PR and QS meet at A. PQ and SR are produced to meet at B, and $AQBR$ is a cyclic quadrilateral.



Prove that BS is perpendicular to PR.

$\angle PQS = 180° - \angle BQS$] straight angles – supplementary.
$\angle PRS = 180° - \angle PRB$]

also $\angle PQS = \angle PRS$ (subtend same arc)

$\Rightarrow 180° - \angle BQS = 180° - \angle PRB$

$\Rightarrow \angle BQS = \angle PRB$

Also $\angle BQS + \angle PRB = 180°$ (opp angles in cyclic quad)

$\Rightarrow 2\angle PRB = 180°$

$\angle PRB = 90°$

i.e. $BS \perp PR$.

Q.E.D.